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Writing about the Problem-Solving Process to Improve Problem-Solving Performance

Problem solving is generally recognized as one of the most important components of mathematics. In *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics emphasized that instructional programs should enable all students in all grades to "build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving" (NCTM 2000, p. 52). But how do students become competent and confident mathematical problem solvers?

Problem solving is a process. It involves a set of skills that can and should be taught. In *How to Solve It*, George Pólya (1945) first discussed the value of using heuristics in problem solving. He used the term *heuristics* to refer to the mental processes of problem solving. Pólya outlined four executive processes used by mathematicians when solving problems: (1) understand the problem; (2) devise a plan; (3) carry out the plan; and (4) look back at the problem.

Understanding the problem involves determining what the problem is seeking and what information and conditions are provided. Devising a plan involves finding a strategy that may help solve the particular problem. This phase may involve analyzing the relationship between the given data and the unknown quantity, thinking about similar problems, making lists, drawing diagrams, or modifying the original problem so that it can be related to a known problem. Carrying out the plan involves following the solution procedure and checking it for flaws. If the initial plan is not correct, it may have to be modified or discarded, and a new plan may have to be developed. Looking back involves making sure that the solution is reasonable, reflecting on the strategies used and whether other strategies might have been used, and comparing the problem or solution with other problems or solutions.

Even when teachers model these executive processes, teaching problem-solving strategies is difficult. Problem solving is a process that takes students time to learn and develop. What can teachers do to help students develop this process?

Mathematicians and mathematics educators generally agree that learning to use the executive processes of problem solving will improve students' problem-solving performance. Writing is a valuable way for students to reflect on mathematical concepts and solidify their understanding of them (NCTM 2000). Thus, writing about the executive processes of problem solving and the problem-solving process in general may not only improve students' problem-solving performance, but it may also help students more clearly understand the problem-solving process. Furthermore, it may help students better organize their thought processes when they attempt to solve problems.

The purpose of the study described in this article was to determine whether writing about the executive processes of problem solving and the problem-solving process in general improved the problem-solving performance of beginning algebra students more than just learning to use the executive processes of problem solving without doing any required writing activities. Forty-two beginning algebra students—all of whom were taught by the same teacher, studied the same topics and completed the same homework assignments and examinations—participated in this study, which lasted one semester at a community college in southeastern Michigan.

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METHOD

This study used a pretest-posttest design with a treatment group and a control group. The treatment

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group consisted of twenty-two students; the control group had twenty students. Each group learned about the executive processes of problem solving. On many example problems, the teacher modeled these processes, using appropriate strategies to solve each problem.

In addition to the regular homework problems, the teacher assigned approximately five nonroutine homework problems to each group each week. These nonroutine problems required more analysis than textbook exercises. They generally could not be solved by applying an algorithm that had been discussed in class. They were more challenging than typical textbook exercises in beginning algebra.

Students wrote about the given information, strategies attempted, and difficulties encountered

The investigator obtained the nonroutine problems from various sources, and the teacher graded them on a scale of zero to four. The teacher gave no points for a problem—

- if the student made no attempt to solve the problem;
- if the data in the problem were simply recopied, but no other work was completed; or
- if some work was completed, but the student seemed to lack any understanding of the problem.

The teacher awarded one point on a problem—

- if the student started toward a solution that reflected some understanding of the problem, but that start could not lead to a solution; or
- if the student used an inappropriate strategy, did not carry it out, and did not attempt another strategy.

Two points were awarded—

- if the student used an inappropriate strategy to get an incorrect answer, but the work showed some understanding of the problem; or
- if the student used an appropriate strategy but did not carry it out far enough or implement it correctly and the student obtained an incorrect answer or no answer.

The teacher gave three points for a problem

- if the student implemented an appropriate strategy and proceeded toward a solution that was incorrect only because the student misunderstood or ignored a condition of the problem; or
- if the student properly applied appropriate strategies but either gave no answer or gave an incorrect answer.

The teacher awarded four points for a problem—

- if the student selected and implemented appropriate strategies and gave the correct answer; or
- if the student selected and implemented appropriate strategies, but a copying or computation error prevented the student from obtaining the correct answer.

For each of these nonroutine problems, each student in the treatment group wrote one or two paragraphs about his or her use of executive processes to solve the problem. They wrote about the given information, strategies attempted and other strategies that might have been attempted, and difficulties that they encountered. They compared the problem with similar problems that they had previously solved. Even if students in the treatment group could not solve a particular problem, they still wrote a paragraph or two about it and focused on the difficulties that they were having and the strategies that they attempted. The control group was not required to complete any writing activities related to the problem-solving process.

When writing about the executive processes of problem solving, students did not have to address each executive process in order. They were not required to answer specific questions about the executive processes of problem solving but instead were writing in general about executive processes and the whole problem-solving process. In particular, many students focused on difficulties that they encountered while they attempted to solve the problems, strategies they tried, and similarities of the problems to previously solved or attempted problems. The teacher responded to all the students' writing. Some examples of students' writing are given below.

Problem

The numbers 6 and 25 are two of my locker number's twelve factors. What is my locker number?

Student A: We are given that the number has twelve factors, and 6 and 25 are two of them. I'll try to make a list of the factors that I know that this number must have. Since 6 is a factor, 2 and 3 must be factors; since 25 is a factor, 5 must be a factor; and of course 1 is a factor. That's six factors so far. $2 * 5, 3 * 5, 2 * 3 * 5, 2 * 5 * 5, 3 * 5 * 5, 2 * 3 * 5 * 5$ must also be factors. That's twelve factors altogether: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, and 150. These are all of the factors of 150, so my locker number is 150.

I made a list to try to come up with the correct answer. This reminded me of another problem where I was trying to count the number of ways of doing something. In that problem, I tried to make a list of all of the possible ways that objects could be selected. In this problem, I tried to list all of the different ways that prime factors could be multiplied to come up with other factors.

Student B: Since 6 and 25 are both factors of my locker number, and they have no prime factors in common, 6 * 25 = 150 must be a factor of my locker number. I'm now going to list the factors of 150: 1, 150, 2, 75, 3, 50, 5, 30, 6, 25, 10, 15. 150 has twelve factors, so it must be the correct answer.

Based on the given information, I guessed that the correct answer might be 150. I then checked to see how many factors 150 had, and I came up with twelve. I actually used two strategies here: guess and check and make a list. I used this guess-and-check strategy previously when I was trying to come up with four natural numbers whose product was equal to their sum. In that problem, I tried different sets of four numbers until I zeroed in on a solution.

Student C: I know that 6 and 25 are factors. Factors of 6 and 25, like 2, 3, and 5, will also be factors of my locker number. 1 is also a factor. That's 6 factors so far. I'm stuck here. How do I get the rest of the factors? If I multiply some of these factors, will I get more factors? How do I know which factors to multiply? I need some help.

The investigator conducted *t*-tests to compare mean pretest scores, mean homework scores for the last thirty nonroutine homework problems, and mean posttest scores between the treatment and control groups. The results follow.

RESULTS

Learning to use the executive processes of problem solving and writing about these executive processes and the problem-solving process in general for each nonroutine problem that students attempted to solve resulted in gains in the problem-solving performances of beginning algebra students. These gains were greater than those of students who learned to use executive processes without doing any required writing activities. **Table 1** summarizes the results of this study. The mean posttest score for the treatment group (32.4) was significantly higher than the mean posttest score for the control group (26.0), whereas no significant difference occurred in pretest means. The mean score for the treatment group on the final thirty nonroutine homework problems (78.6) was also significantly higher than that of the control group (69.0). More than 75 percent of the students in the treatment group wrote that they enjoyed the writing activities related to problem solving. Furthermore, more than 80 percent of students in the treatment group wrote that the writing activities helped them become better problem solvers.

TABLE 1

Differences in Mean Pretest, Homework, and Posttest Scores between the Treatment and Control Groups

	Treatment ($n = 22$)	Control ($n = 20$)	t
Pretest	21.2 (53%)	20.3 (50.75%)	0.40
Homework	78.6 (65.5%)	69.0 (57.5%)	2.06*
Posttest	32.4 (81%)	26.0 (65%)	2.43*

*t is significant if $t > 2.021$

CONCLUSIONS

Research has shown that the use of executive processes and heuristic strategies can improve students' problem-solving performance (Schoenfeld 1985). Furthermore, writing about a mathematical concept helps students organize their thought processes about that concept, focus on difficult points, and more clearly understand the concept (NCTM 2000). Thus, writing about the executive processes of problem-solving and the problem-solving process in general may not only improve students' problem-solving performance but may also help students more clearly understand the problem-solving process.

Both the treatment and control groups improved their problem-solving performances during the semester. However, students in the treatment group improved their problem-solving performances by a greater amount than did students in the control group. Writing about the executive processes of problem solving, difficulties encountered, alternative strategies that they might have used, and the problem-solving process in general apparently helped students in the treatment group learn to use executive processes more quickly and more effectively than students in the control group.

More than 80 percent of the students wrote that the writing activities helped them become better problem solvers

Furthermore, these writing activities may help students better organize their thought processes when they attempt to solve problems, and they may help students have less difficulty when they begin solving challenging problems. When students explain mathematical concepts in writing, they learn to clearly and accurately communicate; and by thinking about these concepts and writing about them in their own words, they may better understand and better remember them. In short, the writing activities may give students the support they need to learn to use executive processes effectively.

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